On the correct modeling of semiconductor optical amplifier RIN and phase noise for optical phase shift keyed communication systems

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Abstract: Phase modulation schemes are attracting much interest for use in ultra-fast optical communication systems because they are much less affected by fiber nonlinearities than conventional modulation formats. Semiconductor optical amplifiers (SOAs) can be used to amplify and process phase modulated signals. However, existing SOA nonlinear phase noise (NLPN) models are simplistic and, sometimes, inaccurate. It is, therefore, important to correctly model their behavior since NLPN is the main drawback in these applications. In this paper we show that a more accurate model can be used leading to simple nonlinear noise expressions at the SOA output of differential phase shift keying systems. To demonstrate the utility of this model, we have used it to calculate the optical signal to noise ratio penalties introduced by a power booster SOA and the first inline amplifier of a 40 Gb/s NRZ-DQPSK single channel link. The model parameters have been estimated from measurements taken of a commercial SOA.

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References
1. Introduction

Semiconductor Optical Amplifiers (SOAs) are a very attractive low cost alternative to Erbium Doped Fiber Amplifiers (EDFAs) since they have smaller size, present a wider bandwidth, higher power efficiency and can be easily integrated [1]. Constant envelope modulation formats, and in particular RZ- and NRZ-DPSK [4], are among the most promising candidates for SOA-based high bit rate systems because their resilience to fiber non-linear effects [12] and to bit pattern effects [2].

Gain saturation in SOAs introduces an excess nonlinear phase noise (NLPN) that is very detrimental in differential phase modulation systems since the information is encoded in the phase of the optical wave. The phase noise behavior of saturated SOAs in DPSK and DQPSK systems has been analyzed in several papers [5,6] and different NLPN compensation proposals have been reported (see [6] and references therein). The NLPN model used in these papers has the advantage of being analytically tractable and, therefore, simplifies considerably the design of NLPN compensation devices. It also provides an easy estimation of the NLPN-induced Optical Signal to Noise Ratio (OSNR) penalty [5] for DPSK applications both for RZ and NRZ formats. However this model is not very accurate. It does not consider either the noise generated inside the SOA, does not properly account for the scattering losses and assumes an input noise signal that is spectrally flat (white) and whose probability density function (PDF) is Gaussian to account for the finite input OSNR. In this paper we show that these assumptions are not always correct since the SOA input noise is hardly ever spectrally flat. We also show that an existing computationally simple and more accurate noise model can be used that leads to simple NLPN expressions at the SOA output of a constant envelope DPSK or DQPSK link.

2. An approximate analytical NLPN model

The model used in [3,5,6] relies on an approximate relationship between the optical wave’s phase and the integral of the saturated gain [11]:

\[ \phi_{out}(t) = \phi_{in}(t) + \alpha \int_0^L g_s(z) dz \] (1)

where \( \phi_{out}(t) \) is the phase of the SOA output, \( \phi_{in}(t) \) is the phase at the input, \( \alpha \) is the line-width enhancement factor, \( g_s(z) \) is the net saturated gain coefficient, \( z \) is the longitudinal coordinate and \( L \) is the SOA length. In this model, a spectrally flat (white) input noise whose PDF is Gaussian [6] models the input signal relative intensity noise (RIN) that accounts for the finite input OSNR. This term induces nonlinear carrier and gain fluctuations along the SOA which produce the nonlinear phase modulation (phase fluctuations) described by Eq. (1). There are several approximations in this model: Eq. (1) does not take into account the SOA amplified spontaneous emission (ASE). However, ASE noise clamps the amplifier’s gain at low input powers and is an internal noise source for NLPN effects. It does not correctly account for the scattering losses either, which do play a very important role in the NLPN behavior of the SOA, as we will see in this paper, and, finally, the input SOA noise cannot always be accurately described by a random function that is, at the same time, spectrally flat (white noise) and whose PDF is Gaussian. This assertion will also be justified in this paper.

This model, thus, only accounts for the NLPN components due to carrier fluctuations induced by the relative intensity noise (RIN) of the optical input signal and, therefore, its predictions will only be valid when this noise term is dominant and spectrally flat.

This model is a very useful tool to have a first order approximation of the NLPN behavior of an in-line and a preamplifier SOA and develop compensation techniques since it provides an approximate semi-analytical expression. However, it cannot be applied to power boosters.
and may yield to inaccurate results in other applications and, in our opinion, should be refined by the use of a more sophisticated model.

3. A more accurate numerical NLPN model

The noise properties of a constant envelope DPSK or DQPSK signal which is amplified by a saturated SOA can be described by a more accurate model [7]. This model includes Langevin noise functions whose correlation relations are chosen so that the results of this semi-classical approach agree with the fully quantum approach. Despite its apparent complexity, the required computation time is low. This model predicts the NLPN at the SOA output caused by a monochromatic optical input wave. It takes into account both the RIN and phase noise of the input signal and also the noise generated inside the SOA. The expressions derived for the RIN and phase noise are also valid in constant envelope DPSK and DQPSK systems. The reason underlying this fact is that the carrier population depends only on the light intensity. Since differential phase shift keying is ideally a constant envelope modulation (this is a good approximation in NRZ systems and only a rough one in RZ systems) the SOA input light power remains constant. The symbol which is being amplified automatically provides a natural decomposition basis for the noise: in-phase noise (also known as amplitude or RIN noise) and quadrature noise (also known as phase noise). Integral expressions for both these noise components were derived in [7]. The phase noise component limits the performances of constant envelope DPSK and DQPSK systems.

This model, which will not be fully described here, remains, therefore, valid and the different RIN and phase noise components can be calculated by evaluating a set of integral expressions (Eqs. (15)-(24)). The basic model equations in the spectral domain (ω) are [7]:

\[
\frac{\partial}{\partial z} \left( \frac{\partial \rho}{\partial \rho_s} \right) = - \frac{g_s \rho_s^2}{1 + \rho_s^2 + i \omega \tau} \frac{\partial \rho}{\partial \rho_s} + N_\rho(\omega, z) \tag{2}
\]

\[
\frac{\partial \delta \phi}{\partial z} = \frac{\alpha g_s \rho_s^2}{1 + \rho_s^2 + i \omega \tau} \frac{\partial \rho}{\partial \rho_s} + N_\phi(\omega, z) \tag{3}
\]

where \( z \) is the distance from the SOA input, \( \rho_s(z) \) is the signal field amplitude, \( \delta \rho(z) \) the amplitude noise (amplitude fluctuations induced by the Langevin force \( N_\rho(\omega, z) \)) which is assumed to be small compared to \( \rho_s(z) \), \( \delta \phi(z) \) the phase noise (phase fluctuations induced by the Langevin force \( N_\phi(\omega, z) \)) which is also assumed to be small compared to the field phase \( \phi_s(z) \), \( g_s(z) \) the SOA saturated gain, \( \tau \) is the carrier lifetime and \( \alpha \) the linewidth enhancement factor. The Langevin forces account for field fluctuations due to spontaneous emission, carrier noise and another term arising from their interaction. The analytical expressions are [7]:

\[
N_\rho(\omega, z) = \frac{1}{2} \frac{\tau}{1 + \rho_s^2 + i \omega \tau} F_\rho(\omega, z) + \frac{1}{2} \left[ \frac{f(\omega, z)}{\rho_s e^{i \phi}} + \frac{f^*(\omega, z)}{\rho_s e^{-i \phi}} \right] \tag{4}
\]

\[
N_\phi(\omega, z) = - \frac{\alpha}{2} \frac{\tau}{1 + \rho_s^2 + i \omega \tau} F_\phi(\omega, z) + \frac{1}{2i} \left[ \frac{f(\omega, z)}{\rho_s e^{i \phi}} - \frac{f^*(\omega, z)}{\rho_s e^{-i \phi}} \right] \tag{5}
\]

The terms \( f(\omega, z) \) and \( F_\rho(\omega, z) \) are also Langevin forces in the spectral domain. They are the Fourier transforms of \( f(t, z) \) and \( F_\rho(t, z) \) whose correlation relations are given by [7]:

\[
\langle f^*(t, z) f(t', z') \rangle = \frac{\hbar \omega}{P_{\text{sat}}} g n_s \delta(t-t') \delta(z-z') \tag{6}
\]

\[
\langle f(t, z) f(t', z') \rangle = \langle f^*(t, z) f^*(t', z') \rangle = 0 \tag{7}
\]
\[
\left\{ F_s(t,z)F_s(t',z') \right\} = \frac{a}{\tau A_c} \left[ \xi g_0 + g_s + aN_s(1+\xi) + g_s\rho_s^2(2n_{sp}-1) \right] \delta(t-t') \delta(z-z')
\]  
(8)

\[
\left\{ F_s(t,z)f(t',z') \right\} = -\frac{a\rho_s e^{i\theta_s} g_s n_{sp}}{A_c} \delta(t-t') \delta(z-z')
\]  
(9)

where \( n_{sp} \) is the SOA inversion factor, \( \delta \) is Dirac’s function, \( a \) stands for the differential gain coefficient, \( A_c \) for the effective cross-section area of the active region, \( \xi \) is a parameter describing the noise of the SOA current source (it is equal to 1 when it exhibits shot noise behavior) and \( N_s \) is the carrier density at transparency. The terms \( \rho_s(z) \) and \( g_s(z) \) represent the unperturbed, (that is to say, unrelated to noise) distributions along the SOA of the square root of the intensity and the saturated gain. They are derived setting the time derivative to zero in the following nonlinear equations that describe the field and gain along the SOA [7]:

\[
\frac{\partial E}{\partial z} = \frac{1}{2} g_s (1-i\alpha) - \gamma_{sc} \]  
(10)

\[
\frac{\partial g}{\partial t} = \frac{g_s - g_s}{\tau} \frac{g_s|E|^2}{\tau}
\]  
(11)

\[
E = \rho_s(z) \exp[i\theta_s(z)]
\]  
(12)

where \( E \) is the square root of the optical power divided by the square root of the SOA saturation power, \( g_s \) is the non-saturated gain, and \( \gamma_{sc} \) the waveguide scattering losses.

Both the relative intensity (RIN) noise and phase noise spectra contain four terms [7] that describe the contributions of the input signal RIN and phase noise (denoted by \( R_{s,co}(\omega) \) and \( \Phi_{s,co}(\omega) \)), the spontaneous emission (denoted by \( R_{sp,co}(\omega) \) and \( \Phi_{sp,co}(\omega) \)), the carrier noise (denoted by \( R_{c,co}(\omega) \) and \( \Phi_{c,co}(\omega) \)) and the cross correlation between the spontaneous emission and the carrier noise (denoted by \( R_{c,sp}(\omega) \) and \( \Phi_{c,sp}(\omega) \)) [7]. They have been calculated in [7] solving Eq. (2) and Eq. (3) taking into account Eq. (4) and Eq. (5) and the correlation functions Eqs. (6)-Eq. (9). Their expressions are given bellow for completeness:

\[
R_s(\omega) = |H(0)|^2 \text{RIN(}\omega,z=0)\]
(13)

\[
R_{sp}(\omega) = \frac{h\omega_b}{\rho_{sat}} \int_0^L |H(z)|^2 \frac{2g_s n_{sp}}{\rho_s^2} dz
\]  
(14)

\[
R_c(\omega) = \frac{h\omega_b}{\rho_{sat}} \int_0^L |H(z)|^2 \frac{\xi g_0 + g_s + aN_s(1+\xi) + g_s\rho_s^2(2n_{sp}-1)}{(1+\rho_s^2)^2 + (\omega\tau)^2} dz
\]  
(15)

\[
R_{c,sp}(\omega) = -\frac{h\omega_b}{\rho_{sat}} \int_0^L |H(z)|^2 \frac{2(1+\rho_s^2)^2 g_s n_{sp}}{(1+\rho_s^2)^2 + (\omega\tau)^2} dz
\]  
(16)

\[
\Phi_s(\omega) = S_{sp}(\omega,z=0) + \frac{\alpha^2 G}{4} |H(0)-1|^2 \text{RIN'}(\omega,z=0) - \frac{2\alpha}{\rho_s(0)} \text{Re}[\left(H(0)-1\right)S_{sp,sp}(\omega,z=0)]
\]  
(17)
\[ \varphi_p^c (\omega) = \frac{\hbar \omega}{4P_{sat} \omega} \int_0^z \left( \alpha^2 |H(z)| - 1 \right|^2 + 1 \frac{2g_r n_{sp}}{\rho_i^2} dz \]  \hspace{1cm} (18)

\[ \varphi_s^c (\omega) = \frac{\hbar \omega}{4P_{sat} \omega} \int_0^z \alpha^2 |H(z)| \left( g_0 + g_s + aN_s \left( 1 + \xi \right) + g_s \rho_i^2 \left( 2n_{sp} - 1 \right) \right) \left( 1 + \rho_i^2 \right)^2 \left( \omega \tau \right)^2 \]  \hspace{1cm} (19)

\[ \varphi_{s,sp}^c (\omega) = \frac{\hbar \omega}{P_{sat} \omega} \int_0^z \text{Re} \left[ \frac{H(z) (1 - H^*(z))}{1 + \rho_i^2 + \omega \tau} \right] \alpha^2 g_s n_{sp} dz \]  \hspace{1cm} (20)

\[ RIN^c = R_s + R_{sp} + R_s + R_{s,sp} \]  \hspace{1cm} (21)

\[ S_{\delta \phi}^c = \varphi_s^c + \varphi_{s,sp}^c + \varphi_s^f + \varphi_{s,sp}^f \]  \hspace{1cm} (22)

\[ \tilde{H}(z) = \left[ \frac{1 - r \left( 1 + \rho_i^2 \right)}{1 + \rho_i^2 + \omega \tau} \right]^{1 \text{/lin}} \]  \hspace{1cm} (23)

\[ H(z) = \frac{\tilde{H}(L)}{H(z)} \]  \hspace{1cm} (24)

where \( P_{sat} = A \hbar \omega / \alpha \tau \), \( S_{\delta \phi}(\omega, z = 0) \) represents the input signal phase noise spectrum, \( S_{\delta \phi, \delta \rho}(\omega, z = 0) \) is the cross correlation power spectrum of the amplitude and phase noise and \( r = \gamma_{\phi, \rho} / g_0 \).
Fig. 2. Saturated gain coefficient distribution along the SOA. The normalized input powers are 0.001 (blue line), 0.01 (red line), 0.1 (green line) and 0.5 (magenta line).

Fig. 3. Spectral power density of the RIN noise components. The normalized input power is 0.1. The input RIN is assumed to be equal to 0.
Fig. 4. Spectral power density of the phase noise components. The normalized input power is 0.1. The input phase noise is assumed to be equal to 0.

4. NLPN behavior of an SOA power booster

We now investigate the performance of a saturated SOA power booster in a 40 Gb/s single-channel NRZ-DQPSK link. Since we are dealing with a single channel link, in order to minimize the NLPN introduced by the SOA, an ideal optical band-pass filter has been supposed to be placed at its output. The bandwidth of this filter is such that it eliminates the noise high-frequency components that are outside the signal bandwidth. A reasonable value for a 40 Gb/s NRZ-DQPSK link is 20 GHz [14] and, therefore, we will determine the NLPN spectral power density in this frequency range around the optical carrier frequency. Notice that the approximate analytical model fails to describe its behavior since we are assuming that the light source has negligible RIN and phase noise and, therefore, the NLPN is generated totally inside the SOA.

Table 1. SOA model parameters. \( g_0 \) is the unsaturated gain coefficient, \( G_0 \) the total gain, \( \xi \) the current source noise parameter, \( \alpha \) the linewidth enhancement factor, \( \tau \) the carrier lifetime, \( P_{\text{sat}} \) the saturation power, \( L \) the cavity length, \( \gamma_{\text{sc}} \) the scattering loss coefficient and \( r = \gamma_{\text{sc}} / g_0 \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>( G_0 )</td>
<td>20 dB</td>
</tr>
<tr>
<td>( g_0 )</td>
<td>9500 m(^{-1})</td>
</tr>
<tr>
<td>( \xi )</td>
<td>1</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>2.5</td>
</tr>
<tr>
<td>( P_{\text{sat}} )</td>
<td>1.9 mW</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.7 ns</td>
</tr>
<tr>
<td>( L )</td>
<td>920 μm</td>
</tr>
<tr>
<td>( \gamma_{\text{sc}} )</td>
<td>( 4.5 \times 10^3 ) m(^{-1})</td>
</tr>
<tr>
<td>( r )</td>
<td>0.4737</td>
</tr>
</tbody>
</table>

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The model parameters were determined in [8,13] for the commercially available bulk InGaAsP/InP Kamelian SOA, model OPA-20-N-C-FA and their values are listed in Table 1. $\zeta$ is the current source noise parameter used in the Langevin correlation functions [7].

Equations (4), (5) and (6) are easily integrated using the Runge-Kutta algorithm to determine the unperturbed field and saturated gain distributions along the SOA. The normalized power is shown in Fig. 1 and the saturated gain coefficient in Fig. 2.

In Fig. 3 and Fig. 4 the RIN and phase noise components due to spontaneous emission noise, carrier noise and cross correlation noise components are shown. The total RIN and phase noises have also been plotted. Figure 5 shows the total phase noise and a Lorentzian fit (a similar figure fit can be obtained for the RIN noise spectrum). Notice that the SOA output noise (which will be the input noise of the first inline amplifier) cannot be described by a white (spectrally flat) random function as assumed in [3,5,6]. The simulations results are very sensitive to some parameter changes as we will show later. The spectral power density spectrum is very sensitive to even relatively small changes in $r$ (this parameter is equal to zero in the analytical model [3,5,6]) since it strongly affects the shapes of all phase noise components. The effect of just a 10% increase in $r$ on the total SOA output NLPN is shown in Fig. 5.

5. SOA power booster optical signal to noise ratio

We will now calculate the phase standard deviation and the OSNR at the SOA amplifier output. The phase noise spectral power density can be very accurately described by the following analytical expression:

![Fig. 5. Spectral power density of the total phase for $r = 0.4737$ (blue) and $r = 0.5211$ (red). Their Lorentzian approximations are plotted in green. The normalized input power is 0.1.](image)

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where $A$ is the phase spectral power density at the carrier frequency ($\omega = 0$), $B$ is the phase spectral power density in the flat side of the spectrum, $2C$ is the transmitted signal bandwidth, $\Delta\omega_l$ is the full width half maximum of the Lorentzian component and $R_{EC}(\omega)$ is the rectangular function that describes the flat part of the spectrum. The phase noise autocorrelation function analytical expression can be derived by calculating the inverse Fourier transform of Eq. (7) yielding:

$$R_{\phi\phi}(\tau) = \frac{A-B}{2}\Delta\omega_0 \exp(-\Delta\omega_0 |\tau|) + \frac{2BC}{\sqrt{2\pi}} \text{Sinc}\left(\frac{C\tau}{\pi}\right)$$

(26)

where the function $\text{Sinc}(\tau)$ is defined as $\text{Sinc}(\tau) = \sin(\tau)/\tau$. The phase noise variance is obtained by letting $\tau = 0$ in Eq. (26) [9]. We get:

$$\sigma_{\phi\phi}^2 = (A-B)\frac{\Delta\omega_0}{2} + \frac{2BC}{\sqrt{2\pi}}$$

(27)

Fig. 6. Phase noise standard deviation for input powers ranging from 0.01 to 0.1.

<table>
<thead>
<tr>
<th>Table 2. A, B and C constant values for Fig. 3(a).</th>
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<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
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We have used Eq. (9) to evaluate the phase standard deviation at the SOA booster output for normalized input powers ranging from 0.01 to 0.1. The results have been plotted in Fig. 6. The values of the constants A, B and C related to Fig. 5 (red line) are shown in Table 2.

Some recent papers seem to suggest that the PDF of a DPSK link can be approximated by a Gaussian distribution [10] and, therefore, a BER expression can be deduced. However, to the best of our knowledge [14], the PDF of a NRZ-DQPSK random data stream in the presence of phase noise has not been assessed yet and so, all we can do is to calculate the OSNR [5,15] (the only noise we are considering is the NLPN):

\[
\text{OSNR} = 10 \log_{10} \frac{\pi^2}{8 \sigma_{\phi}^2} \quad (28)
\]

If we take the most unfavorable case of Fig. 6 (\(P_{\text{in}} = 0.01\)), Eq. (28) yields an OSNR equal to 31.1 dB which is a large figure. In the next section we will study to what extent this OSNR is degraded by a single inline SOA amplifier.

The NLPN behavior is very sensitive to the values taken by the parameters \(a\), \(r\) and is still much more pronounced to small changes in \(r\). Our simulations show that SOAs with high scattering losses and longer cavities have smaller output OSNRs. This will be discussed in more detail in the following section.

![Simulated NRZ-DQPSK single channel link.](image)

**Fig. 7. Simulated NRZ-DQPSK single channel link.**

### 6. NLPN behavior of an SOA in-line amplifier

We will now study the performances of a saturated SOA in-line power amplifier in the same 40 Gb/s single-channel NRZ-DQPSK previously considered. This amplifier introduces two additional NLPN terms: a first one, due to the coupling between the input RIN noise (generated by the power booster), \(R_{\text{in}}(\omega)\), and the output phase noise and a second one related to the input phase noise, \(\Phi_{\text{in}}(\omega)\). We assume that the input RIN and phase noise are uncorrelated to simplify the calculations, slightly overestimating the output noise [7]. The simulation results prove that the in-line SOA significantly reduces the OSNR.

In Fig. 7, we show the single channel DQPSK link that we have simulated. The relative input power of the inline SOA will be the same one considered for the power booster (\(P_{\text{in}} = 0.01\)). We evaluate how sensitive the OSNR penalty is to changes in the different parameter values and different normalized input powers.
Fig. 8. OSNR at the output of the power-booster and the in-line amplifier as a function of $\alpha$. The other parameters take their nominal values.

Fig. 9. OSNR at the output of the power-booster and the in-line amplifier as a function of $\tau$. The other parameters take their nominal values.
Fig. 10. OSNR at the output of the power-booster and the in-line amplifier as a function of $\tau$. The other parameters take their nominal values.

Fig. 11. OSNR at the output of the power-booster and the in-line amplifier as a function of the normalized input power.
7. Conclusions

In this paper, we have shown that the commonly used NLPN SOA model in constant envelope DPSK systems does not always correctly describe their behavior. We have seen that it does not predict the performances of a saturated power booster, and that the input noise of the inline amplifiers is not always well described by a random function that is, at the same time, spectrally flat (white) and Gaussian as assumed when using it. Moreover, the internal noise should be taken into account and the scattering losses do have a strong influence in the OSNR degradation of inline amplifiers. We propose the use of an already existing accurate model, based on a more detailed description of the physical phenomena taking place inside the saturated SOA. This model can be applied to constant envelope PSK systems since the intensity of the light propagating along the SOA is ideally constant. We have also shown how this more accurate and computationally simple model can be used to calculate the OSNR degradation that takes place in a single channel 40 Gb/s NRZ-DQPSK system.

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