

# Michelson interferometer vibrometer using self-correcting synthetic-heterodyne demodulation

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Synthetic-heterodyne demodulation is a useful technique for dynamic displacement and velocity detection in interferometric sensors, as it can provide an output signal that is immune to interferometric drift. With the advent of cost-effective, high-speed real-time signal-processing systems and software, processing of the complex signals encountered in interferometry has become more feasible. In synthetic heterodyne, to obtain the actual dynamic displacement or vibration of the object under test requires knowledge of the interferometer visibility and also the argument of two Bessel functions. In this paper, a method is described for determining the former and setting the Bessel function argument to a set value, which ensures maximum sensitivity. Conventional synthetic-heterodyne demodulation requires the use of two in-phase local oscillators; however, the relative phase of these oscillators relative to the interferometric signal is unknown. It is shown that, by using two additional quadrature local oscillators, a demodulated signal can be obtained that is independent of this phase difference. The experimental interferometer is a Michelson configuration using a visible single-mode laser, whose current is sinusoidally modulated at a frequency of 20 kHz. The detected interferometer output is acquired using a 250 kHz analog-to-digital converter and processed in real time. The system is used to measure the displacement sensitivity frequency response and linearity of a piezoelectric mirror shifter over a range of 500 Hz to 10 kHz. The experimental results show good agreement with two data-obtained independent techniques: the signal coincidence and denominated n-commuted Pernick method. © 2015 Optical Society of America

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## 1. INTRODUCTION

Optical interferometers can be used as sensitive displacement and velocity measurement systems. The basic principle is that an applied stimulus causes a phase shift between two light beams, which are combined, converted to an electrical signal. This signal has a nonlinear relationship to the phase shift; thus, relatively complex signal processing techniques need to be applied to obtain an output signal that is proportional to the stimulus. Common techniques include pseudo-heterodyne [1], passive and active homodyne [2] and synthetic heterodyne [3]. The latter technique is particularly useful, since it also eliminates interferometric drift, whereby slow variations in the optical paths due to environmental influences cause the interferometry sensitivity to slowly vary. In all of this work, the demodulation process was implemented using analog circuits. In [4] we used digital implementation (signal acquisition using an analog-to-digital converter) and signal processing of synthetic-heterodyne demodulation to detect the displacement

of a 100 Hz vibrating reflective diaphragm in a low-finesse Fabry–Perot interferometer. A current modulated narrow-linewidth 1550 nm laser was used as the optical source. At that time, equivalent visible lasers were not commercially available. However, the detected displacement was only approximately estimated, and the sensitivity was not optimized because of lack of knowledge, in particular, of the argument of the Bessel functions present in the theoretical expression for the demodulated output.

In this paper, implementation of synthetic-heterodyne demodulation is described, which uses two additional quadrature oscillators, applied to a visible laser Michelson interferometer. We use it to measure the frequency response, up to 10 kHz, and linearity of a mirror shifter. The experimental results show good agreement with data obtained using two alternative schemes: the signal coincidence method (SCM) [5] and the denominated n-Commuted Pernick method (n-CPM) [6]. We show how the interferometer visibility can be measured and the Bessel function argument set by using an appropriate value

of the laser diode modulation current, which is automatically updated to account for possible changes in the interferometer mean path difference and the sensitivity of the laser optical frequency to its bias current.

## 2. THEORY AND IMPLEMENTATION

A schematic of the sensor system under consideration is shown in Fig. 1. The laser used is a 658 nm wavelength stabilized single longitudinal mode visible laser diode (Ondax TO-658-PLR35) having a linewidth of 50 MHz and a side-mode suppression ratio  $> 30$  dB. The laser was operated at a bias current of 65 mA at which the output power is 20 mW. The laser current is amplitude modulated by a  $f_0 = 20$  kHz sine wave using a 50 kHz bandwidth current driver. The collimated laser beam is input to a Michelson interferometer. The object under test is a piezoelectric mirror shifter (PMS) consisting of a mirror mounted on a piezoelectric actuator (Piezomechanik mirror shifter ST-35), which is driven by a 1 MHz bandwidth high-voltage amplifier (Thorlabs HVA200).

The recombined reference and object light beams are detected by a photodiode (Thorlabs SM05PD2A) and transimpedance amplifier (Stanford low-noise current amplifier SR570), the output voltage of which is passed through a 100 kHz anti-aliasing lowpass filter before acquisition using a 16 bit resolution 250 kHz sampling rate data acquisition (DAQ) module (National Instruments USB-6211), which is also capable of signal generation. The acquired signal is then processed in real-time using LabVIEW. In addition to laser power modulation, the current modulation also causes modulation of the optical frequency. Since the modulation current amplitude  $I_m$  is small (typically  $< 5$  mA) compared to the laser bias current, amplitude modulation induced variations in the laser power are assumed to be negligible compared to the photodiode signal component induced by the interferometer. The detected interferometric voltage signal at the DAQ input is given by

$$v(t) = A + AV \cos[C \cos(2\pi f_0 t) + \theta(t)], \quad (1)$$

where  $A$  is the average voltage,  $V$  the interferometer visibility, and  $\theta(t)$  is the phase shift due to the interferometer path difference.

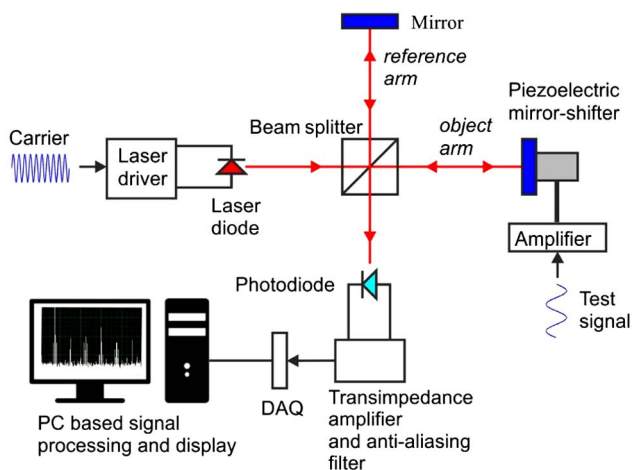


Fig. 1. Experimental setup.

$$\theta(t) = \phi(t) + \varphi(t), \quad (2)$$

where  $\varphi(t)$  is a slowly varying phase shift due to changes in the interferometer medium refractive index caused by fluctuations in the ambient conditions.  $\phi(t)$  is the dynamic phase shift between the reference and test object beams, induced by the PMS dynamic displacement  $x(t)$ , and is given by

$$\phi(t) = \frac{4\pi}{\lambda} x(t), \quad (3)$$

where  $\lambda$  is the source wavelength. A typical measured interferometric signal, after acquisition, and its spectrum is shown in Fig. 2. The sampling points are joined by straight-line segments. However, the signal is very similar to the form of Eq. (1) for a sinusoidal displacement. The quantization noise is less than the noise floor of the current amplifier.

Equation (1) can be expanded into a DC term plus an infinite series of carriers located at integer multiples of  $f_0$ , multiplied by either  $\cos \theta(t)$  or  $\sin \theta(t)$ :

$$\begin{aligned} v(t) = A + AV \bigg\{ & J_0(C) \cos \theta(t) \\ & + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(C) \cos(4\pi k f_0 t) \cos \theta(t) \\ & - 2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(C) \cos[2\pi(2k+1)f_0 t] \\ & \times \sin \theta(t) \bigg\}, \quad (4) \end{aligned}$$

where  $J_i(C)$  are Bessel functions of the first kind of order  $i$  with argument  $C$  given by

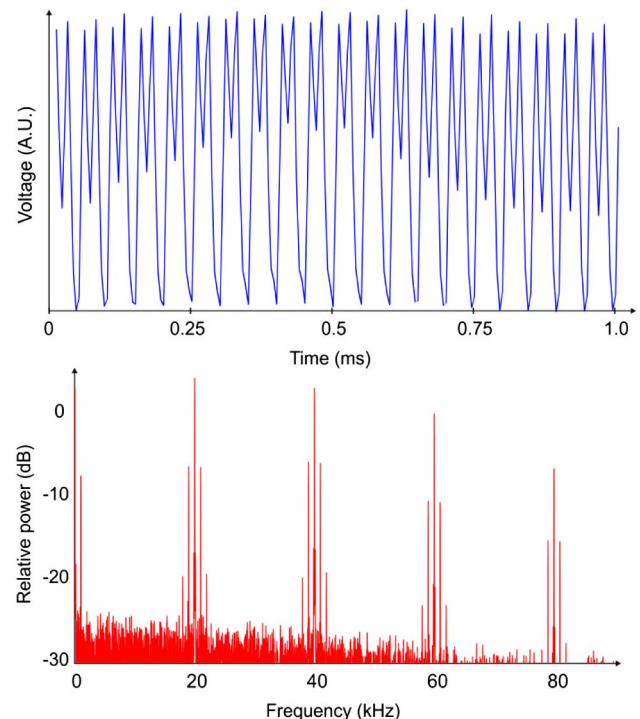


Fig. 2. (a) Typical measured interferometric signal after acquisition and (b) its spectrum. The vibration frequency is 1 kHz.

$$C = \frac{4\pi L}{c} \frac{dv}{di} I_m, \quad (5)$$

where  $L$  is the nonzero mean path difference,  $c$  the speed of light in free space, and  $dv/di$  the sensitivity of the laser optical frequency with respect to current (approximately 1.5 GHz/mA for the laser used in this work). In synthetic-heterodyne demodulation, two terms in Eq. (3) with  $\cos \phi(t)$  and  $\sin \phi(t)$  can be processed to obtain a signal proportional to the time rate of change of  $\theta(t)$ . We consider the terms in Eq. (4) at  $f_0$  and  $2f_0$ , given by

$$S_1(t) = -2AVJ_1(C) \cos(2\pi f_0 t) \sin \theta(t), \quad (6)$$

$$S_2(t) = -2AVJ_2(C) \cos(4\pi f_0 t) \cos \theta(t). \quad (7)$$

Multiplying  $S_1$  and  $S_2$  by local oscillators  $\cos(2\pi f_0 t + \beta)$  and  $\cos(4\pi f_0 t + \beta)$ , respectively, which have an unknown phase  $\beta$  relative to the detected interferometric signal in Eq. (1), and lowpass filtered (bandwidth of 7.5 kHz), to reject terms at  $2f_0$  and  $4f_0$  gives

$$S_3(t) = -AVJ_1(C) \sin \theta(t) \cos(\beta), \quad (8)$$

$$S_4(t) = -AVJ_2(C) \cos \theta(t) \cos(\beta). \quad (9)$$

Differentiating  $S_3$  and  $S_4$  gives

$$S_5(t) = -AVJ_1(C) \dot{\theta}(t) \cos \theta(t) \cos(\beta), \quad (10)$$

$$S_6(t) = AVJ_2(C) \dot{\theta}(t) \sin \theta(t) \cos(\beta). \quad (11)$$

Multiplying  $S_3$  by  $S_6$  and  $S_4$  by  $S_5$  gives

$$S_7(t) = -(AV)^2 J_1(C) J_2(C) \dot{\theta}(t) \sin^2 \theta(t) \cos^2(\beta), \quad (12)$$

$$S_8(t) = (AV)^2 J_1(C) J_2(C) \dot{\theta}(t) \cos^2 \theta(t) \cos^2(\beta). \quad (13)$$

Subtracting  $S_7$  from  $S_8$  gives

$$S_9(t) = (AV)^2 J_1(C) J_2(C) \dot{\theta}(t) \cos^2(\beta). \quad (14)$$

If instead local oscillators  $\sin(2\pi f_0 t + \beta)$  and  $\sin(4\pi f_0 t + \beta)$  are used, we get a signal

$$S_{10}(t) = (AV)^2 J_1(C) J_2(C) \dot{\theta}(t) \sin^2(\beta). \quad (15)$$

Adding  $S_9$  and  $S_{10}$  gives an output signal

$$S_o(t) = (AV)^2 J_1(C) J_2(C) \dot{\theta}(t), \quad (16)$$

which is independent of  $\beta$ . Since the ambient fluctuations are very slow,  $\dot{\phi}(t) \ll \dot{\phi}(t)$ :

$$S_o(t) \approx (AV)^2 J_1(C) J_2(C) \dot{\phi}(t). \quad (17)$$

To determine  $\dot{\phi}(t)$  requires knowing the values of  $AV$  and  $J_1(C)J_2(C)$ . In the case of small displacement, the maximum deviation of the interferometer signal in Eq. (1) from its average value is guaranteed for  $C \geq \pi$ , in which case  $AV$  can be easily obtained as half the difference between the maximum and minimum of  $v(t)$ . It is desirable to choose some  $C \geq \pi$  such that the magnitude of  $|J_1(C)J_2(C)|$  is large as well as relatively insensitive to  $C$ . As seen from the plot of  $J_1(C)J_2(C)$  shown

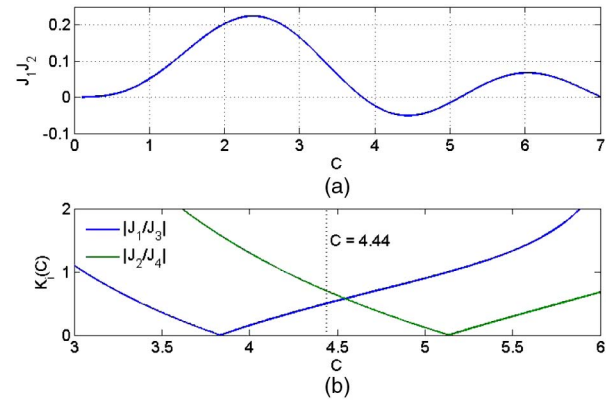


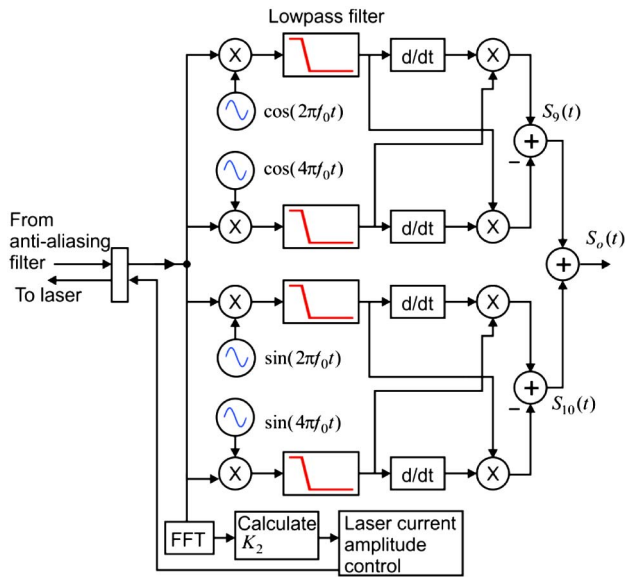
Fig. 3. (a)  $J_1(C)J_2(C)$  and (b) Bessel function ratios versus  $C$ .

in Fig. 3(a), the first maximum/minimum occurs at  $C = 4.44$  at which  $|J_1(C)J_2(C)| = 0.051$ . Also the slope of the  $J_1(C)J_2(C)$  characteristic is zero at this point. The next maximum has a greater magnitude but to reach its  $C$  value requires a larger modulation current amplitude. If the mean interferometer path difference and laser frequency sensitivity are known, then  $C$  can be set by choosing an appropriate modulation current and using Eq. (5); however, both of the latter quantities are only approximately known, and, in particular, the laser frequency sensitivity is difficult to measure and depends on bias current and temperature. It is desirable, then, to use an automatic procedure such that a particular value of  $C$  can be maintained. The ratio of the magnitudes of successive terms in Eq. (4) proportional to  $\cos \theta(t)$  or  $\sin \theta(t)$  (except for the term including  $J_0(C)$ ) are given by

$$K_i(C) = \left| \frac{J_i(C)}{J_{i+2}(C)} \right|, \quad (18)$$

which only depends on  $C$ . This ratio is shown in Fig. 3(b) for  $i = 1$  and 2. In contrast to  $K_1(C)$ ,  $K_2(C)$  decreases monotonically for  $C$  ranging from 0 to the first 0 of  $J_2(C)$ , which is equal to 5.14. In order to set  $C$  equal to the desired value of 4.44, the modulation current amplitude is increased from 0 to a value at which  $K_2 = 0$ .  $K_2$  is measured by calculating the square root of the ratio of the powers of the terms in  $v(t)$  centered at frequencies  $2f_0$  and  $4f_0$ , using its fast Fourier transform (FFT) averaged over a time span of a few seconds. However, the power of  $v(t)$  centered at frequencies  $2f_0$  and  $4f_0$  can become close to zero, depending on the value of the random phase shift; then,  $K_1(C) = 0.5$  can be calculated from  $f_0$  and  $3f_0$ , which represent the same set of  $C$  equal to 4.44 in Fig. 3(b). The ambiguity of Eq. (18) can be avoided by observing maximum of the spectrum magnitude for each frequency  $nf_0$  ( $n = 1, 2, 3, \dots$ ). According to [7], when the maximum is switching between  $3f_0$  and  $4f_0$  by the random phase shift, then  $C$  is monotonic within a given limited range of 4.2 to 5.13. This is also convenient to adjust  $C$  to 4.44 from Eq. (18). A schematic of the signal-processing algorithm and laser control is shown in Fig. 4.

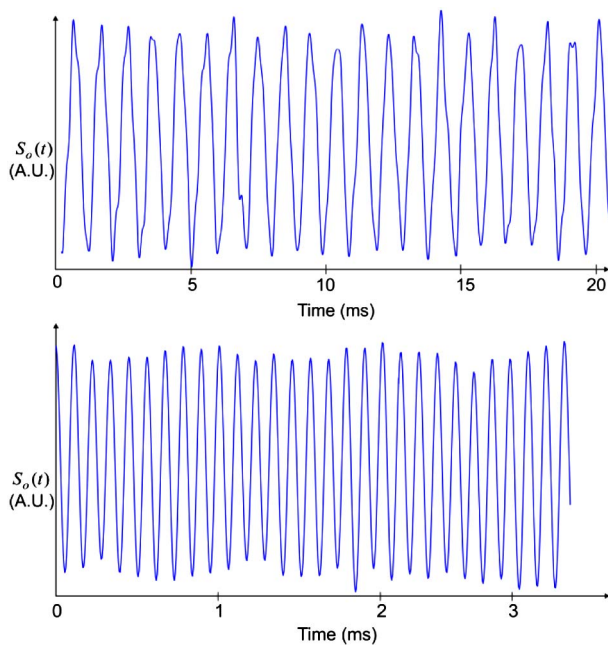




**Fig. 4.** Synthetic-heterodyne demodulation and laser control block diagram.

### 3. EXPERIMENT

The system was used to measure the PMS sensitivity (nm/volt) small-signal frequency response and linearity. The mean path difference was approximately 2 cm. A small-signal sinusoidal voltage at frequency  $f$  was applied to the PMS leading to an unknown induced displacement amplitude phase amplitude  $x_0$ . Typical demodulated 1 and 9 kHz vibration signals are shown in Fig. 5.



**Fig. 5.** Demodulated 1 and 9 kHz vibration signals.

Since  $AV$  and  $C$  are known,  $x_0$  can be obtained from Eqs. (3) and (17) as

$$x_0 = \frac{s_0 \lambda}{8\pi^2 f (AV)^2 J_1(C) J_2(C)}, \quad (19)$$

where  $s_0$  is the amplitude of the demodulated sinusoidal output signal. There is some distortion, which may be due to the introduction of a frequency-dependent DC phase shift by the transimpedance amplifier/antialiasing filter to the detected interferometric signal.

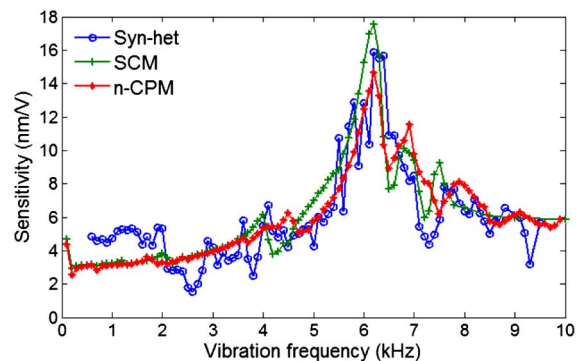
By way of comparison, the measured response was compared with measurements obtained using two independent measurement procedures: (a) the SCM, whose application is detailed in ISO 16063-41 [8], is an applicable modification of the standard for primary vibration calibration ISO 16063-11 [9], which is described in detail in [5]; (b) the denominated n-CPM [6]. This latter method uses the magnitude spectrum of the photodetected signal from the interferometer with no carrier modulation applied. In the case of sinusoidal vibration, the photodetected signal (neglecting slow and small ambient fluctuations in phase) is given by

$$w(t) = A + AV \cos[\phi_0 \cos(2\pi ft)]. \quad (20)$$

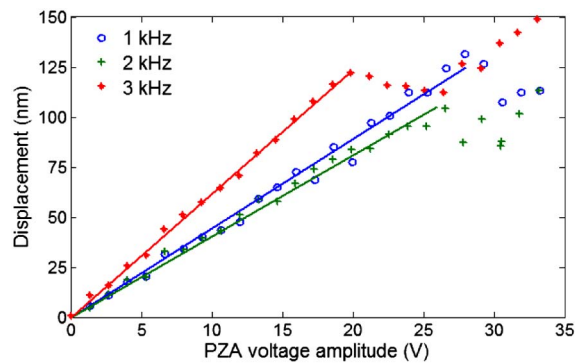
If  $V_1, V_2, V_3, \dots$  are the harmonic magnitudes of the fundamental and the higher-order harmonics of  $w(t)$ , and the harmonic with maximum magnitude  $V_n$ , then  $\phi_0$  can be determined from

$$\phi_0^2 = \frac{4(n+1)(n+2)(n+3)|V_{n+2}|}{(n+3)|V_n| + 4(n+2)|V_{n+2}| + (n+1)|V_{n+4}|}, \quad (21)$$

and then  $x_0$  can be obtained directly from Eq. (2). For the small-signal voltage applied to the PMS, the first harmonic had the maximum amplitude. The measured small-signal sensitivity is shown in Fig. 6; for the voltage amplitudes used, there was negligible harmonic distortion indicating operation in the linear regime. The experimental results using SCM and n-CPM agree well with the synthetic-heterodyne data. The PMS exhibits strong resonance at a frequency of approximately 6.3 kHz. The measured PMS response at various frequencies is shown in Fig. 7, showing good linearity over a 0–20 range of drive voltage amplitudes.



**Fig. 6.** PMS sensitivity frequency response.



**Fig. 7.** PMS displacement versus applied voltage.

#### 4. CONCLUSIONS

A self-correcting digital synthetic-heterodyne demodulation scheme was implemented in real time using fast signal acquisition and processing. The system was successfully used to measure the frequency response of a mirror-shifter, and the results show good agreement with measurements taken using the n-CPM and SCM techniques.

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